Constrained Structural Optimization for Aeroelastic Requirements

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In carrying out aeroelastic structural optimization analyses, the use of fixed vibration mode shapes may prove unsatisfactory since variations in mass and stiffness distributions result in changing modes. A method of structural optimization is presented in which the aeroelastic structural modal shapes are represented as functions of mass distribution, structural stiffness, and aerodynamic forces. The optimality condition is the minimum weight, which satisfies given structural strength constraints and precludes the onset of flutter over the airplane speed-altitude domain. Modern finite element and displacement mthods are used to describe the aeroelastic equations of motion in matrix form. Aerodynamic forces are given in quasi-static lift distribution vs displacement matrix form, and modal shapes are the solution of a matrix eigenvalue problem. The onset of flutter is that point at which diagonalization proves unattainable.

Nomenclature

$[A]$ = aerodynamic force matrix
a_i, b_i = coefficients of the characteristic equation of $[L]$ and its reduced approximation respectively
$[K]$ = assembled stiffness matrix of the structure
$[K_F]$ = combined stiffness matrix of fixed structural elements
$[K_i],[m_i]$ = unitized element-stiffness and inertia matrix for the variable elements which, when multiplied by the scaling functions $f_i(\mu_i)$, $g_i(\mu_i)$, yield the actual stiffness and inertia matrices
$[L]$ = inverse mass-stiffness-aerodynamic matrix to be diagonalized
$[M_F]$ = combined inertia matrix for nonvariable elements and nonstructural mass items
$[m[$ = consistent inertia matrix
Q = dynamic pressure parameter
r = number of variable elements
$\{x\}$ = nodal degree-of-freedom modal vector
y_i = the ith iterate of arbitrary vector $\{y\}$ by the matrix $[L]$
θ = flutter phase angle
λ = eigenvalue of $[L]$
μ_i = ith element mass
$\bar{\mu}_i$ = prescribed minimum value of ith variable element mass
ω = oscillatory angular frequency

Introduction

THE structural engineer is traditionally motivated by the need to provide positive, quasi-static, strength margins for the combined g's, airloads, and the thermal environment associated with critical flight maneuvers. In achieving this objective, his primary concern is that of providing structural integrity along the major structural load paths. He therefore evolves minimum sizes for the main structural members defining these load paths, regardless of whether or not he employs structural optimization techniques in the process.

On the other hand, the aeroelastic engineer is essentially concerned with providing adequate torsional rigidity to prevent divergence, and aeroelastic torsional-to-bending frequency ratios which preclude flutter within the flight regime. From this point of view, the structural components which are considered critical by the aeroelastician and the structural designer are not necessarily the same.

In view of the complexity of the analyses required to keep the computational effort within practical levels, it is of great importance, for practical aircraft problems, to outline efficient optimization procedures and to limit the application of the procedure to only the components most critical from an aeroelastic standpoint. The flutter optimization procedure described and demonstrated herein is based upon just such a philosophy and, accordingly, we have incorporated the structural strength constraints in the form of minimum practical gages and lower bounds on the sizes of major structural members.

The flutter optimization technique is based upon a finite-element idealization of the structure wherein the structural operators in the aeroelastic equations of motion are represented by the assembled stiffness matrix. Following Turner,[1] we require that the total structural weight be stationary with respect to mass-parameter control variables and aerodynamic-response state variables, where satisfaction of the aeroelastic equations is enforced through the use of Lagrangian multipliers. However, aside from this common aspect of the two approaches, our optimization formulation and numerical techniques differ from Turner's in the following major respects:

1) Following Pines,[2] we simplify the aerodynamic representation by employing accurate experimentally- or analytically-based, quasic-static, lift distributions for deformed structures at each instant. Thus, lift damping due to bending and other unsteady phenomena are neglected in the optimization process for purposes of defining the onset of flutter. Using this approach, 250 comparisons of flutter predictions were made with the results of experiments by McDonnell[3] for 100 separate primary surface cases, and 100 comparisons were made for about 30 control surface cases. It was found that flutter predictions employing quasi-static, aerodynamic parameters were quite accurate for primary surfaces and reasonably ac-

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ceptable for control surfaces (at least comparable to classical results). This simplification permits us to deal with real, rather than complex, variables in the aeroelastic equations, thereby considerably reducing the numerical computations required in the optimization procedure.

2) Employing the above aerodynamic simplifications, we have found that the onset of flutter is characterized by both the coalescence of two modal frequencies and by the coalescence of the *corresponding mode shapes.* From a mathematical point of view, this implies that the unsymmetrical eigenvalue problem, represented by the flutter equations, becomes *defective in eigenvectors* at the flutter point. Employing this new condition, we have been able to develop a *single* iteration process leading simultaneously to a minimum-mass solution and the associated flutter frequency. This approach is to be distinguished from that employed by Turner[1,4] wherein optimality solutions for a whole sequence of fixed frequencies and a graphical solution for the minimum-mass distribution were required. In our approach, the frequency is automatically adjusted during the iteration process and we therefore avoid the solution of a sequence of optimization problems, with the result that computational labor and computer running time are greatly reduced.

3) In most methods used in structural dynamic design problems, fixed modes are employed as a data base. Usually these are zero-speed, ground-vibration modes of the initial structure. This is done in order to avoid repeating large-scale finite-element structural analyses. In the present method, the displacement vector for the current structure is used directly, and, hence, the mode shapes that are used in the analyses are the ones that would be encountered at the flight condition under study.

4) The numerical techniques employed in our optimization procedure were developed with a view towards minimizing the computational difficulties encountered in problems involving large matrices. Thus, stiffness matrices are employed in the formulation without requiring that they be inverted, and linearized forms of the optimality equations are handled by the matrix-solver technique of triangular-factorization, again relaxing the stringent precision requirements associated with large matrix sizes.

With regard to problem-size considerations, a new matrix-reduction technique has recently been developed by Ojalvo and Newman[5] for accurately extracting the lower modes of large eigenvalue systems by means of an improved version of the basic Lanczos Method.[6] A least-squares eigenvalue method based on an approximation to the Cayley theorem is also described and recommended.

No attempt is made to detail the generation of aerodynamic influence coefficients in this paper. One-dimensional lifting line theory, two-dimensional membrane theory, and three-dimensional steady-state aerodynamic matrix methods are available in the literature. What is required for us in this method is a matrix representation of the aerodynamic force, or moment, as a function of the surface deflection or local slope.

To demonstrate the practical significance of the above consideration, we have documented herein the application of our flutter optimization technique to a six-degree-of-freedom primary wing model, with variable spar cap and skin gage parameters, under two sets of conditions. In the first case it was assumed that, in order to maintain adequate wing-bending strength, a minimum of 70% of the initial-design spar cap material was to be retained. Subject to this constraint, a flutter-free, minimum-weight design which is 15% lighter than the initial design was achieved. In the second case, no structural constraints were imposed. A weight reduction of 48% was effected but, because of the unrealistic absence of structural constraints, the spars were driven down almost to the vanishing point whereas the skin gages stabilized themselves at values which prevented wing divergence. Obviously, this latter design possesses unreasonably low bending strength and lacks structural integrity.

Technical Discussion

A) Statement of the Problem

We are concerned with the problem of minimizing the weight of a primary-surface aerodynamic structure having a specified geometric form and operating within a given flight regime. The structural elements are assumed to have known material properties and to deform in a linear-elastic manner.

The design is subject to the following constraints: 1) No flutter or divergence instabilities shall take place at specified flight conditions. 2) Gages or size-parameters of chosen structural elements shall not fall below minimum specified values. 3) Nonstructural weights and inertias shall remain invariant during the optimization process. 4) Mach number and altitude are specified.

B) Formulation of the Aeroelastic Equations

Employing a finite element approach and the displacement method of analysis, and aeroelastic equations of motion for steady-state oscillations can be expressed in the matrix form

$$([K] - \omega^2[m] - Q[A])\{x\} = 0 \tag{1}$$

The form of Eq. (1) admits aerodynamic representations of varying degrees of complexity and, in typical flutter analyses, the matrix $[A]$ is usually expressed as a complex function of the reduced frequency parameter, Mach number, and altitude. In this analysis, the matrix $[A]$ is the set of aerodynamic influence coefficients of lift, or moment, given as a linear function of the displacement and independent of frequency.

We assume that the structural elements fall into two categories: 1) Elements which must remain fixed in size. 2) Elements which are permitted to vary, but whose sizes must be equal to, or greater than, some prescribed minimum values specified by the structural analyst.

If we consider that the stiffness and inertia of a general ith variable element are proportional to prescribed functions of the element mass, we may then write

$$[K] = \sum_{i=1}^{r} f_i(\mu_i)[K_i] + [K_F] \tag{2a}$$

$$[m] = \sum_{i=1}^{r} g_1(\mu_i)[m_i] + [M_F] \tag{2b}$$

subject to

$$\mu_i \geq \bar{\mu}_i\ ,\ \ i = 1, 2, ---, r \tag{2c}$$

Alternatively, if we define the initial design mass of the ith variable element as $(\mu_i)_o$, and Eqs. (2c) can be expressed in the form

$$\mu_i \geq R_i(\mu_i)_0\ ;\ i = 1, 2, \ldots, r \tag{2d}$$

where R_i is a structural constraint factor.

In most applications (e.g., thin-walled tubes, rods, panels, and beams, with cross sections definable through a characteristic dimension), the element stiffness and inertia matrices may be considered as directly proportional to the element masses themselves. In fact, this condition can always be approached more and more accurately, if one is willing to create a progressively finer element breakdown of the structure at the expense of increased numbers of degrees-of-freedom.

When this direct proportionality relationship is employed, Eqs. (2) reduce to the simpler form

$$[K] = \sum_{i=1}^{r} \mu_i[K_i] + [K_F] \tag{3a}$$

$$[m] = \sum_{i=1}^{r} \mu_i [m_i] + [M_F] \tag{3b}$$

$$\mu_i \geq R_i(\mu_i)_0 \; ; \; i = 1, 2, \ldots, r \tag{3c}$$

C) The Optimality Equations

Our task is to minimize the total variable structural mass

$$\Omega = \sum_{i=1}^{r} \mu_i \tag{4}$$

subject to the constraints imposed by Eqs. (1) and (3). Introducing the aeroelastic equations via a Lagrange multiplier row vector Γ, we therefore require that the created response functional

$$\psi = \sum_{i=1}^{r} \mu_i + [\Gamma]\left\{\sum_{i=1}^{r} \mu_i([K_i] - \omega^2[m_i]) + [K_F] - \omega^2[M_F] - Q[A]\right\}\{x\} \tag{5}$$

be stationary with respect to variations in the vector

$$\{v\} = \begin{Bmatrix} x_1 \\ \cdot \\ \cdot \\ x_j \\ \cdot \\ \cdot \\ x_n \\ \cdots \\ \mu_1 \\ \cdot \\ \cdot \\ \mu_i \\ \cdot \\ \cdot \\ \mu_r \\ \cdots \\ \omega^2 \end{Bmatrix} = \begin{Bmatrix} \{x\} \\ \cdots \\ \{\mu\} \\ \cdots \\ \omega^2 \end{Bmatrix} \tag{6}$$

subject to the constraints Eq. (3c). Here, $\{x\}$ and ω^2 are, respectively, an eigenvector and an eigenvalue of Eq. (1) for a prescribed flight condition and mass vector $\{\mu\}$.

It is to be noted that, in Turner's development,[1,4] the natural frequency ω^2 is *not* considered as a variational parameter, thereby requiring optimality solutions for a sequence of fixed frequencies and a graphical solution for the minimum total mass distribution. This cumbersome aspect is removed in the present approach and a single iteration process leads simultaneously to minimum total mass and the associated flutter frequency.

Introducing the quasi-static aerodynamic simplifications mentioned earlier, the aerodynamic matrix becomes independent of frequency and involves real variables only. Consequently, when the indicated variational process is carried out, the following system of *real*, coupled, nonlinear equations in $\{x\}$, $\{\mu\}$, and ω^2 is found to govern the mass-minimization problem

$$[B(\{\mu\}, \omega^2)]\{x\} = \{0\} \tag{7a}$$

$$[B(\{\mu\}, \omega^2)]^T\{\bar{x}\} = \{0\} \tag{7b}$$

$$[\bar{x}]([K_i] - \omega^2[m_i])\{x\} = \delta \; , \; i = 1, 2, \ldots, r \tag{7c}$$

$$[\bar{x}][m]\{x\} = 0 \tag{7d}$$

$$\mu_i \geq R_i(\mu_i)_0 \; ; \; i = 1, 2, \ldots, r \tag{8}$$

where

$$[B(\{\mu\}, \omega^2)] = \sum_{i=1}^{r} \mu_i\{[K_i] - \omega^2[m_i]\} + [K_F] - \omega^2[M_F] - Q[A] \tag{9}$$

$\{\bar{x}\}$ = an eigenvector of the transpose (or adjoint) eigenvalue problem, $[B]^T\{\bar{x}\} = 0$

δ = a constant

The underlying significance of the above equations is as follows:

1) To achieve a minimum-weight solution, the difference between a pseudo-strain energy density and a pseudo-kinetic energy density [this energy term being defined by the left side of Eq. (7c)] must be the same for each variable element throughout the structure. This energy may be viewed as the virtual work per unit of element mass which the modal forces perform in traveling through the adjoint modal-displacement field.

2) Equation (7d) is a biorthogonality condition between the modal vector $\{x\}$ and its adjoint counterpart $\{\bar{x}\}$. This is the *flutter condition,* since it can only occur when two roots of Eq. (1) and the corresponding modal vectors coalesce. Mathematically speaking, Eq. (7d) implies that the system be comes defective in eigenvectors[7] at the flutter point.

From a conceptual point of view, the energy criterion given by Eq. (7c) is the nonconservative, dynamic-system counterpart of that employed by Venkayya et al.[8] for the static strength optimization of structures wherein, for weight minimization, the ratio of strain energy in an element to its energy capacity are required to be the same throughout the structure. In fact, the similarity between the two sets of criteria suggests that both might be used advantageously to achieve simultaneous optimization for stress compliance and aeroelastic requirements.

D) Method of Solution

If we denote the number of degree-of-freedom by n, the eigenvalue problem posed by Eq. (7a) provides n independent equations in $\mu_1, \mu_2, \ldots, \mu_r, \omega^2$ and $n - 1$ components of the modal vector $\{x\}$. The adjoint eigenvalue problem, Eq. (7b), provides only $n - 1$ independent equations, since solutions of Eq. (7a) ensure that the determinant of $[B]^T$ vanishes. Thus, Eqs. (7a–d), subject to the constraints (8), constitute $2n + r$ nonlinear equations in $2n + r$ unknowns, namely, r values of μ_i, ω^2, δ, and $2(n - 1)$ modal vector components. The variable element masses $\mu_1, \mu_2, \ldots, \mu_r$ do not appear in Eqs. (7c), which therefore contain only $2n - 1$ unknowns. Thus, the number of variable elements r must satisfy the requirement $r \leq 2n - 1$ in order to avoid an overprescribed system.

Equations (7) are to be solved for an optimal solution vector

$$\{V^*\} = \begin{Bmatrix} \{x\}^* \\ \cdots\cdot \\ \{\bar{x}\} \\ \cdots\cdot \\ \{\mu\}^* \\ \cdots\cdot \\ (\omega^2)^* \end{Bmatrix} \tag{10}$$

where

$$\{\mu\}^* = \begin{Bmatrix} \mu_1 \\ \vdots \\ \mu_i \\ \cdot \\ \cdot \\ \mu_r \end{Bmatrix}^* \geq \begin{Bmatrix} R_1(\mu_1)_0 \\ \vdots \\ R_i(\mu_i)_0 \\ \cdot \\ \cdot \\ R_r(\mu_r)_0 \end{Bmatrix} \tag{10a}$$

represents the vector of optimum element mass distribution, such that

$$\Omega = \Omega_{\min} = \sum_{i=1}^{r} \mu_i^*$$

and (ω^*) is the optimum flutter frequency.

Numerical solutions for a prescribed flight condition are obtained via the following iteration procedure:

1) Starting with a base design, for which the initial element mass distribution is $\{\mu\}^{(1)} = \{(\mu)_o\}$, solve Eq. (7a) and its adjoint, Eq. (7b), for the corresponding eigensolutions $\{x\}^{(1)}$, $\{\bar{x}\}^{(1)}$, and $(\omega^2)^{(1)}$. This yields the first trial vector

$$\{V^{(1)}\} = \begin{Bmatrix} \{x\}^{(1)} \\ \cdots\cdots \\ \{\bar{x}\}^{(1)} \\ \cdots\cdots \\ \{\mu\}^{(1)} \\ \cdots\cdots \\ (\omega^2)^{(1)} \end{Bmatrix} \tag{11}$$

2) Let $\{\bar{V}^{(1)}\} = \{V^{(1)}\} + \{\Delta V^{(1)}\}$ where $\{\Delta V^{(1)}\}$ is a correction vector. Substitute $\{\bar{V}^{(1)}\}$ into the nonlinear system defined by Eqs. (7) and (8), and render the resulting formulation linear in the $\{\Delta V^{(1)}\}$ terms. Solve the resulting system of linear algebraic equations for $\{\Delta\mu\}^{(1)}$. This vector, when premultiplied by an appropriate scaling matrix $[\alpha]^{(1)}$, yields a correction to the initial mass vector $\{\mu\}^{(1)}$, i.e., we set

$$\{u\}^{(2)} = \{\mu\}^{(1)} + [\alpha]^{(1)}\{\Delta\mu\}^{(1)} \tag{12}$$

3) Steps 1 and 2 are repeated, using progressively updated mass distribution vectors, $\{\mu\}^{(1)}, \{\mu\}^{(2)}, \{\mu\}^{(3)}, \ldots$, until a convergence criterion is met, indicating sufficiently small changes in each element of the mass distribution vector. The resulting vector represents $\{\mu\}^*$, the optimum mass distribution vector.

The program logic is constructed so that each time the eigenvalue problem is solved (Step 1), a search is made for that pair of roots which are closest in magnitude. The lower root of this pair is then employed in subsequent calculations. This technique permits us to follow a flutter-critical solution branch during the optimization process. In addition, if complex conjugate roots appear, indicating that we have passed the flutter point, the mass vector $\{\mu\}$ is scaled upward until real eigenvalues of Eqs. (7a) are restored. Since it is important to follow the modal branches that are closest to coalescence, we define a measure of modal parallelism, called the flutter phase angle θ, which measures the angle between two modes whose eigenvalues are close in numerical magnitude. We have

$$\theta = \cos^{-1}\left(\frac{\lfloor x_1 \rfloor \{x_2\}}{|\{x_1\}|\,|\{x_2\}|}\right)$$

At flutter, this angle is zero.

The structural constraints of Eqs. (8 or 10a) are enforced by slack functions, γ_i, such that

$$[\mu_i - R_i(\mu_i)_0] - \gamma_i^2 = 0 \tag{13}$$

With regard to numerical methods, the program employs a determinantal root-finding routine to obtain eigenvalues and Cayley-matrix products to construct the eigenvectors.

In Step 2 of the computational procedure, which involves solving a system of linear algebraic equations for the mass vector increment $\{\Delta\mu\}$, a triangular factorization matrix-solver routine similar to Cholesky's scheme is employed. Experience with large matrices has shown that this approach is much more efficient from a computational standpoint than matrix inversion, particularly when the matrix is sparsely-directed, as in the present case.

A Least-Squares Solution of the Multi-Dimensional Flutter Eigenvalue Problem

For extremely large structural assemblages, the computational burden in obtaining eigensolutions of the aeroelastic equations [Eqs. (1)] becomes prohibitive unless special techniques are employed to reduce the problem size in some manner, while still providing a sufficient number of aeroelastic modes for flutter predictions.

A new matrix-reduction process of this type has recently been developed by Ojalvo and Newman[5] for symmetrical eigenvalue problems. The technique is an improved version of the basic Lanczos method,[6] which overcomes the numerical stability and precision problems encountered by Wilkinson[7] and others. It has been employed successfully to obtain preselected numbers of lower vibration modes in structural problems involving several hundred degrees-of-freedom.[9]

An extension of this technique to accommodate the unsymmetrical flutter-eigenvalue problem can be evaluated along with other lower-mode extraction schemes, including the Fox-Kapoor conjugate gradient method[10] for minimization of the Rayleigh quotient. In this study, we describe and recommend a least-squares approach based on the Cayley-Hamilton theorem.

In the conventional approach, it is customary to use the lower frequency normal modes of the aircraft at zero airspeed as generalized coordinates, to compute generalized forces due to oscillatory air forces and to solve a reduced-order eigenvalue problem for a sequence of fixed airspeeds. The critical flutter speed is the lowest speed for which the eigenvalues contain undamped motion. In this approach, we avoid fixed normal modes and directly obtain the reduced-order aeroelastic vibration modes up to the flutter boundary.

The flutter eigenvalue problem is to solve the matrix-vector equation

$$[m]^{-1}([K] - Q[A])\{x\} = \omega^2\{x\} \tag{14}$$

In this form, the equations have the advantage of requiring the inverse of a simple nonsingular inertia matrix, with the disadvantage of yielding the highest (noncritical) frequencies first. Since all the eigenvalues are real and positive below the critical speed, we may subtract any upper bound of the highest frequency, $\bar{\omega}^2$, and rewrite Eq. (14). Let

$$[L] = [m]^{-1}([K] - Q[A]) - \bar{\omega}^2[I] \tag{14a}$$

$$\lambda = \omega^2 - \bar{\omega}^2 \quad \lambda \text{ real and negative)} \tag{14b}$$

then Eq. (14) becomes

$$[L]\{x\} = \lambda\{x\} \tag{14c}$$

To reduce the order of the system, we have recourse to the Cayley theorem. The characteristic equation of the matrix $[L]$ is given by

$$\det|[L] - \lambda[I]| = \sum_{i=0}^{N} a_i\lambda^i \tag{15}$$

where N is the order of $[L]$ and large.

The Cayley theorem states that every matrix satisfies its characteristic equation

$$\sum_{i=0}^{N} a_i[L]^i = 0\ ; \quad a_N = 1 \tag{16}$$

For an arbitrary vector in the N-space, $\{y\}$, we have

$$\sum_{i=0}^{N} a_i[L]^i\{y\} = 0 \tag{16a}$$

For P, an integer less than N, we require a least-squares solution of

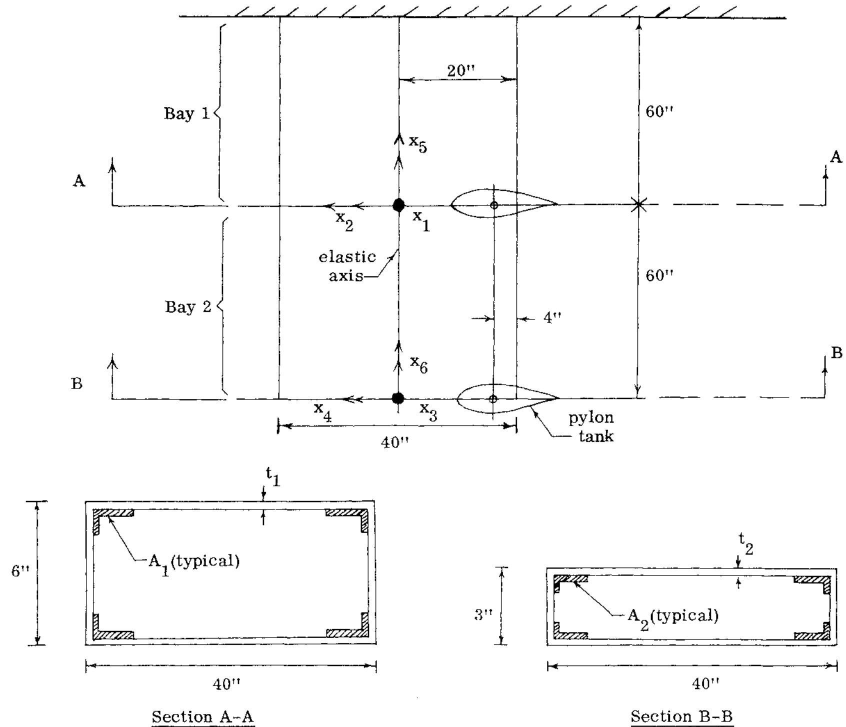

Fig. 1 Six degree-of-freedom rectangular wing model.

$$\sum_{i=0}^{P} b_i [L]^i \{y\} = 0 \ ; \quad b_P = 1 \tag{17}$$

Since the iterates of $[L]^i\{y\}$ are rich in eigenvectors of Eq. (10) corresponding to the smallest values of ω^2, the b_i's obtained from Eq. (17) will be the proper approximation to the a_i, rich in the lower roots. Let

$$\{y\}_i = [L]^i \{y\} \tag{18}$$

Equation (17) may be written

$$[y_{P-1}, y_{P-2}, \cdots, y] \begin{Bmatrix} b_0 \\ \vdots \\ b_{P-1} \end{Bmatrix} = -\{y_P\} \tag{18a}$$

Using either Gram-Schmidt or the Householder orthogonalization procedures, we obtain the least-squares solution

$$\begin{Bmatrix} b_0 \\ \vdots \\ b_{P-1} \end{Bmatrix} = -[T]^{-1}[B]^T\{y_P\} \tag{18b}$$

where $[T]$ is a $P - 1$ triangular matrix and $[B]^T$ is the transpose of the orthogonal decomposition of the first $P - 1$ iterates of $[L]\{y\}$.

If λ_r is a root of the reduced characteristic equation

$$\sum_{i=0}^{P} b_i \lambda_r^{\,i} = 0 \ ; \ b_P = 1 \tag{19}$$

then the corresponding lower required frequency is

$$\omega_r^2 = \bar{\omega}^2 + \lambda_r \tag{19a}$$

The corresponding eigenvector $\{x\}_r$ is given by

$$\{x\}_r = \sum_{i=0}^{P-1} l_i(b, \lambda_r)\{y\}_i \tag{19b}$$

where, by recursion, we have

$$\begin{aligned} l_{P-1} &= 1 \\ l_{P-i} &= \lambda_r l_{P-i+1} + b_{P-i+1} \end{aligned} \tag{19c}$$

This solves the eigenvalue problem.

To obtain the adjoint vectors $\{\bar{x}\}$, a similar series of P adjoint iterates of $\{y\}^T[L]$ are obtained. The same b_i, λ_r coefficients and roots are applicable. The adjoint vectors are given by

$$\{\bar{x}\}_r^T = \sum_{i=0}^{P-1} l_i(b, \lambda_r)\{y^T\}_i \tag{20}$$

where

$$\{y^T\}_i = \{y\}^T[L]^i \tag{20a}$$

Aerodynamic Considerations

The mechanism of basic wing bending-torsion flutter is an instability resonance phenomenon in which energy is taken from the airstream by one mode and is transferred to drive a second mode at their common frequency. No tractable theory exists for describing the true dynamical interaction, in both magnitude and phase, of the aerodynamical forces and the structural deflection. In conventional flutter analysis, aerodynamic forces are computed on the assumption that the structure is oscillating in harmonic motion. Prior to and after the flutter point, this assumption is inaccurate. Only at the flutter point is the assumption of harmonic motion accurate. However, examination of the mechanism of flutter[2] indicates that wing bending-torsion flutter instability is a consequence of the inphase lift and moment and not the result of loss of out-of-phase small damping forces. Consequently, the quasistatic theory yields the major effect at a great reduction in complexity. Of course, for single degree-of-freedom flutter, such as control surface buzz, stall flutter, etc., the above remarks are invalid, and other methods must be employed.

The method proposed here is to obtain the flutter boundary using quasi-steady lifting surface theory. The list distribution will be obtained as a function of the wing deflection form. A detained discription of methods for obtaining quasi-steady aerodynamic influence coefficients as functions of airfoil deflection are beyond the scope of this work. It is sufficient to state that methods exist to cover subsonic and supersonic Mach numbers for various planforms.[11-15]

Sample Problem and Numerical Results

As an application of the foregoing optimization procedure, we consider the simplified unswept, rectangular

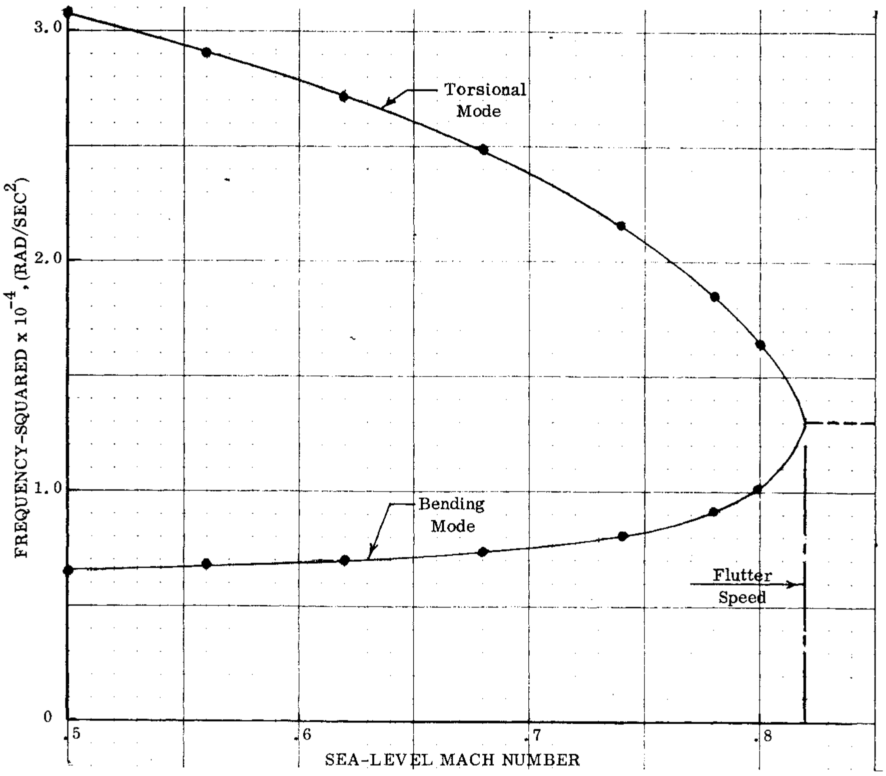

Fig. 2 Variation of frequency with sea-level Mach number for wing model initial design, showing frequency coalescence.

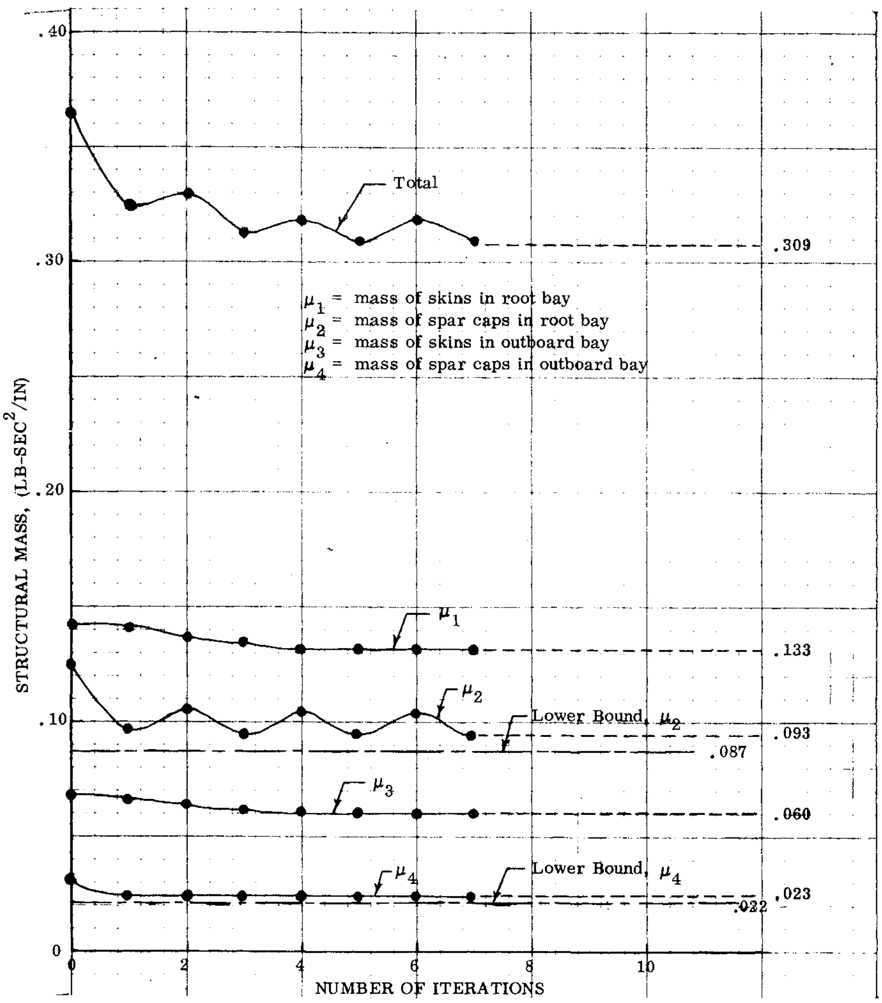

Fig. 3 Wing optimization with structural constraints, M = 0.80, sea level.

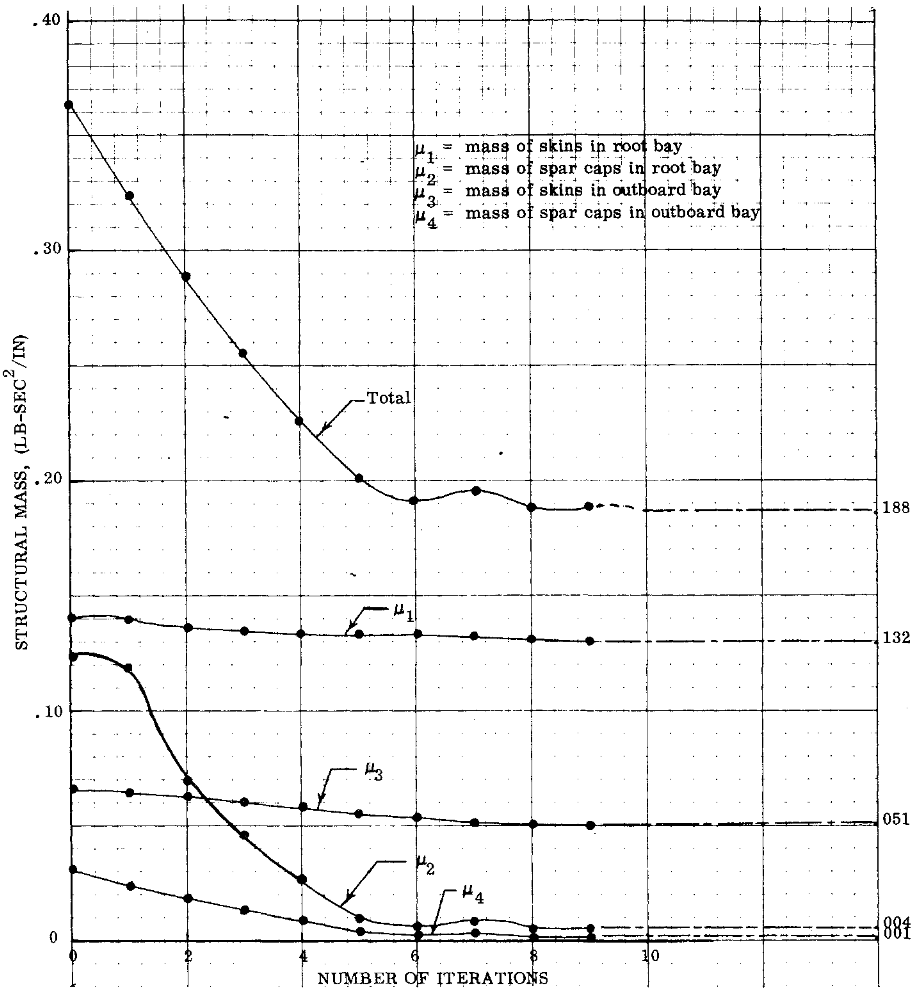

Fig. 4 Wing optimization for M = 0.80, sea level, with no structural constraints.

wing model shown in Fig. 1. The structure is of all-aluminum box-beam construction with variable skin gages (t_1, t_2) and spar cap areas (A_1, A_2) in each of two bays. As shown in the figure, six nodal degrees-of-freedom have been assigned along the semichord elastic axis, consisting of bending deflection, spanwise rotation, and torsional rotation at the end of each bay. Two nonstructural mass items, in the form of pylon tanks, are assumed to be suspended from the wing 16 in. aft of midchord; one at midspan and the other at the wing tip. The spar caps are assumed to possess negligible torsional rigidity, whereas the skins form closed torque cells and also contribute to the wing-bending rigidity.

The initial structural design parameters (before optimization) are shown in Table 1 which includes mass and inertial data for an inboard and an outboard fuel tank. A flutter analysis was first performed on this initial design to find the critical Mach number for sea-level flight, using strip theory and quasi-static aerodynamic coefficients. Figure 2 shows the variation of two modal frequencies with Mach number, specifically those two frequencies which coalesce to produce flutter. The coalescence occurs slightly above M = 0.8 and the corresponding flutter frequency is approximately 18 Hz.

Next, the Mach number is fixed at 0.8 and the optimization procedure was applied. To demonstrate the practical importance of imposing realistic constraints on the member sizes, two separate weight-minimization problems were solved. In the first case, it was assumed that in order to maintain adequate wing-bending strength, a minimum of 70% of the initial-design spar-cap material was to be retained. This implies that the spar-cap structural constraint factors are $R_2 = R_4 = 0.70$ in Eqs. (10a). In the second case, no size constraints were imposed, i.e., R_i = 0; i = 1, 2, 3, 4.

Table 1 Initial structural design—sample problem

Element no.	Description	Thickness, t (in.)	Area per cap, A (in.^{2})	Element mass, μ (lb-sec^2/in.)
1	Skins of bay 1	0.100	...	0.1429
2	Four spar caps in bay 1	...	2.00	0.1242
3	Skins of bay 2	0.050	...	0.0668
4	Four spar caps of bay 2	...	0.50	0.0311
Σ				0.3650

Inboard Tank	*Tip Tank*
$\bar{\mu}_1$ = 0.2671 lb-sec^2/in.	$\bar{\mu}_2$ = 0.0978 lb-sec^2/in.
I_1 = 131.0 lb-in.-sec^2	I_2 = 39.2 lb-in.-sec^2

From a practical viewpoint, convergence to a minimum-weight solution occurs after the seventh iteration. Figure 3 summarizes the trend of structural mass vs iteration number, showing that after seven iterations, the total structural mass has been reduced from its original value of 0.365 lb-sec^2/in. to 0.309 lb-sec^2/in., representing a reduction of approximately 15%. The flutter phase angle at the minimum configuration is 1.44° (very close to the flutter point), and the flutter frequency is approximately 16.7 Hz, as compared to the initial design value of 18 Hz.

Figure 3 shows the skin and spar-cap mass vs the iteration number. It can be seen that the spar-cap masses in the minimum-weight design are slightly higher than the prescribed lower-bound values, thus demonstrating satisfaction of the constraint conditions.

The problem ran approximately 15 sec on a CDC 6600 computer (including compilation time).

Solution for the Unconstrained Minimization Problem

The solution of the problem without size constraints (only necessary that the masses be nonnegative) is summarized in Fig. 4. Convergence to a numerical solution was effected after nine cycles, with a structural weight reduction of approximately 48%. However, because of the unrealistic absence of minimum-size constraints, the spar caps are driven down in weight to the point where they are almost entirely eliminated, whereas the skin gages stabilize at values which prevent wing divergence.

This design would be inadequate from almost any strength or stiffness standpoint, since the skins alone provide little in the way of bending material.

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Minimum Time and Minimum Fuel Flight Path Sensitivity

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The sensitivity of minimum time and minimum fuel flight paths to variations in aircraft parameters in different atmospheric conditions was investigated using the energy state approximation. Numerical results are presented for a typical supersonic aircraft in Standard Day, Hot Day, and Cold Day atmospheres. Minimum time and minimum fuel flight paths in the vertical plane are well documented in the literature. This paper shows how flight time and fuel consumption are affected by changes in thrust, weight, drag coefficients (C_{Do} and C_{Di}), and specific fuel consumption (SFC) in each of three different atmospheric conditions (Standard Day, Hot Day, and Cold Day). For each variation, the effect on performance (flight time or fuel consumption) is determined for the nominal paths. Then for each variation, the flight path is adjusted to be either time optimal or fuel optimal. As a result of this analysis, it was found that flight time and fuel consumption are sensitive to variations in C_{Do}, SFC, and aircraft weight. There is only a slight sensitivity to variations in C_{Di} and no flight time sensitivity to variations in SFC. It was found that a Hot Day atmosphere tended to degrade aircraft performance by increasing flight time and fuel consumption along the flight path, while a Cold Day atmosphere tended to enhance performance. Adjusting the nominal flight paths to be time optimal or fuel optimal for the conditions being considered was found to be desirable for only a limited number of conditions. In a Standard Day atmosphere, the only conditions for which path adjustment significantly improved performance are large thrust reductions and large increases in C_{Do}. In a Hot Day atmosphere, path adjustment improved performance for every variation with the exception of thrust increases and C_{Do} decreases. In a Cold Day atmosphere, path adjustment failed to significantly improve performance for any variation.

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Nomenclature

C_{Do} = drag coefficient at zero lift
C_{Di} = induced drag coefficient
D = drag, lb
E = specific aircraft energy, ft
g = gravitational acceleration, ft/sec^2